Discovering the rules of exponents NAME:

Here we will work through the exponent rules intuitively so they will stick better in our heads. The one thing you must remember is what an exponent means.

You can think of exponents as repeated multiplication. For instance, 2^5 **can be thought of as repeatedly multiplying 2 by itself, 5 times, or** $2*2*2*2*2$**.** We will use this notion to rewrite various expressions involving exponents.

It is important to keep in mind that there are other ways to simplify these expressions (not the least of which is to use our calculator). But we are doing this to help us develop rules that would apply to expressions in general. The first row of each table shows a completed example. Try to leave your expressions in the same form that I have.

1. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the exponent rule in general. The first one is done for you.

Expression to be simplified	**Work it out!**	**End result**
$2^3 * 2^4 \longrightarrow$	$(2*2*2)*(2*2*2*2)$ Notice this is seven 2's. $\longrightarrow$	2^7
$3^4 * 3^1$		
$5^4 * 5^5$		
$7^2 * 7^6$		
$x^m * x^n$ where *x*, *m*, and *n* are just numbers like above		

Does this rule work for $4^3 * 5^2$? Explain. If so, what do you get as the end result?

2. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the exponent rule in general. The first one is done for you.

You'll need to rewrite the part inside the parentheses, and then use the outer exponent to rewrite it again.

Expression to be simplified	Work it out!	End result
$\left(3^2\right)^3$	$(3*3)^3$ which is $(3*3)*(3*3)*(3*3)$ This is six 3's.	3^6
$\left(4^3\right)^4$		
$\left(2^3\right)^3$		
$\left(2^4\right)^2$		
$\left(8^3\right)^1$		
$\left(x^m\right)^n$ where x, m, and n are just numbers like above		

Does this rule work for $\left(3^2\right)^5$? Explain. If so, what do you get as the end result?

3. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the exponent rule in general. The first one is done for you.

These problems also use the commutative property of multiplication (repeatedly) that says the order in which I multiply two numbers does not matter. For instance, $3*4=4*3$**.**

Expression to be simplified	Work it out!	End result
$(3*4)^2$	$(3*4)*(3*4)$ which is $(3*3)*(4*4)$ This is two 3's and two 4's.	3^2*4^2
$(2*5)^3$		
$(6*3)^4$		
$(4*5)^2$		
$(x*y)^n$ where *x*, *y*, and *n* are just numbers like above		

Does this rule work for $(1*13)^3$? Explain. If so, what do you get as the end result?

4. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the exponent rule in general. The first one is done for you.

These problems also use the fact that common factors on top and bottom of a fraction can be cancelled. For instance, $\frac{3*4}{4}=3$ **since the 4's on top and bottom can be thought of as** $\frac{4}{4}$ **which is just 1.**

Expression to be simplified	**Work it out!**	**End result**
$\frac{3^5}{3^3}$	$\frac{3*3*3*3*3}{3*3*3}$ If we cancel the three 3's on bottom with three 3's on top (circled), we're left with two 3's on top.	3^2
$\frac{6^6}{6^3}$		
$\frac{5^2}{5^1}$		
$\frac{8^{10}}{8^5}$		
$\frac{x^m}{x^n}$ where x, m, and n are just numbers like above		

Does this rule work for $\frac{4^2}{4^2}$? Explain. If so, what do you get as the end result?

Does this rule work for $\frac{0^4}{0^2}$? Explain. If so, what do you get as the end result?

5. Complete the table using the idea of repeated multiplication. Then in the final row, use the pattern to write the exponent rule in general. The first one is done for you.

These problems also use multiplication of fractions. Recall we multiply two fractions by multiplying the tops and bottoms separately. For instance, $\frac{3}{4}*\frac{5}{2}=\frac{15}{8}$.

Expression to be simplified	Work it out!	End result
$\left(\frac{3}{2}\right)^3$	$\frac{3}{2}*\frac{3}{2}*\frac{3}{2}$ but using fraction multiplication, we see this as $\frac{3*3*3}{2*2*2}$	$\frac{3^3}{2^3}$
$\left(\frac{5}{6}\right)^4$		
$\left(\frac{1}{2}\right)^2$		
$\left(\frac{2}{9}\right)^5$		
$\left(\frac{x}{y}\right)^n$ where x, y, and n are just numbers like above		

Does this rule work for $\left(\frac{5}{0}\right)^3$? Explain. If so, what do you get as the end result?

6. Complete the table. Then in the final row, use the pattern to write the exponent rule in general. The first one is done for you.

These problems use the exponent rule from #4 to complete the second column. To complete the third column, remember what you get when you divide a number by itself.

Expression to be simplified	**Work it out using the exponent rule from #4 !**	**Work it out using what you know about a number divided by itself**
$\frac{2^3}{2^3}$	2^{3-3} which is just 2^0	This is really $\frac{8}{8}$ which is just 1.
$\frac{5^4}{5^4}$		
$\frac{9^2}{9^2}$		
$\frac{6^5}{6^5}$		
	x^0 where x is just a number like above	

Does this rule work for 0^0? Explain. If so, what do you get as the end result?